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HYPERCUBE AND ITS VARIANTS USING NS2

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Abstract- In this paper, five different variants of interconnection networks such as Hypercube Network, Folded Hypercube Network, Multiply Twisted Hypercube Network, Recursive Circulant Hypercube Network and Multiple Reduced Hypercube Network are illustrated using Network Simulator and various physical and logical factors such as time, size of the packet, type of packet, speed, coverage and packet loss are investigated. This analysis helps in deciding the suitable interconnection network variant for various massive applications.

Key words **-** Interconnection network, variants, physical and logical factors.

I. INTRODUCTION

A Multi-Processor is a computer with more than one central processor. An Interconnection network which is used to link various multi computer processors greatly affects the overall behavior and performance of the entire system model. A wide range of network scales such as degree, connectivity, scalability, diameter, reliability, throughput, speed, network cost, time etc can be used to evaluate the performance of overall interconnection network.

Based on the number of nodes, Interconnection networks are classified into meshes (n×k), hypercube (2n) and star (n!). In an interconnection network, degree related to hardware cost and diameter related to message passing time is correlated with each other. In general, as degree of an interconnection network is increased, diameter is decreased, which can increase throughput in the interconnection network, however, it increases hardware cost with the increased number of pins of the processor when a parallel computer is designed. An interconnection network with less degree reduces hardware cost but increases message passing time, which adversely affects latency or throughput of an interconnection network. Network scales being typically used for comparative evaluation of an interconnection network due to the said characteristic include network cost [4-10] defined as degree x diameter of an interconnection network. By virtue of its merit of easily providing a communication network system required in applications of all kinds.

An interconnection network such as Hypercube can be expressed as an undirected graph, where node indicates a processor and an edge indicates a communication channel among the processors. Hypercube is node-symmetric and edge-symmetric, has a simple routing algorithm with maximal fault tolerance and a simple reflexive system, and also has a merit that it may be readily embedded with the proposed interconnection networks [11,12]. However, it involves weak points that network cost increases due to increase of degree with the increased number of nodes, and that a mean distance between diameter and node is not short as compared with degree. To improve such weak points, Reduced Hypercube [13] that reduced the number of edges of a hypercube interconnection network, Gaussian Hypercube [14], and Exchanged Hypercube [15] have been suggested, and in addition, Crossed Cube [5] that improved diameter of a hypercube interconnection network, Folded Hypercube [6], MRH [7], HFN [4], MRH [1] etc. have been proposed.

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Many interconnection networks that have been proposed until now demonstrated that they have superior network cost to hypercube by reducing just one network scale of degree or diameter of hypercube.

In further section, we illustrate different variants of Hypercube interconnection networks and finally, conclusion is given.

II. TYPES OF INTERCONNECTION NETWORKS

We have different variants of interconnection networks. We consider some of them as given below:

1. Hypercube Network.
2. Folded Hypercube Network.
3. Multiply Twisted Hypercube Network.
4. Recursive Circulant Hypercube Network.
5. Multiple Reduced Hypercube Network.

Hypercube Network:

According to geometry, a Hypercube is n dimensional figure which is analogous to a cube in 3 dimension and a square in 2 dimension. A 2D Hypercube interconnection network which is analogous to a square contains four processors; a processor and a memory module are placed at each vertex of a square. The diameter of the system is the minimum number of steps it takes for one processor to send a message to the other processor that is the farthest away. So, for example, the diameter of a 2-cube is 1. In a hypercube system with eight processors and each processor and memory module being placed in the vertex of a cube, the diameter is 3. In general, a system that contains 2^N processors with each processor directly connected to N other processors, the diameter of the system is N.

One disadvantage of a hypercube system is that it must be configured in powers of two, so a machine must be built that could potentially have many more processors than is really needed for the application.

Construction and different types of Hypercube are shown in Figure (a) [1, 21]. Figure (a) demonstrates how to create a tesseract from a point.

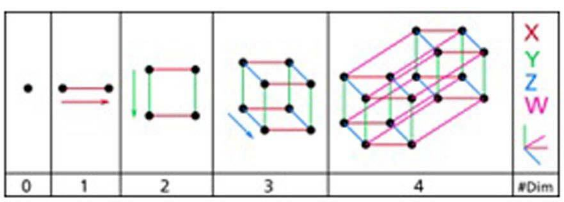

Fig (a) Hypercube types in multiple dimensions

Dimension – 0: A point is a hypercube of dimension zero

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Dimension –1: If one moves this point one unit length, it will sweep out a line segment, which is a unit hypercube of dimension one.

Dimension –2: If one moves this line segment its length in a perpendicular direction from itself; it sweeps out a 2-dimensional square.

Dimension – 3: If one moves the square one unit length in the direction perpendicular to the plane it lies on, it will generate a 3-dimensional cube.

Dimension – 4: If one moves the cube one unit length into the fourth dimension, it generates a 4-dimensional unit hypercube (a unit tesseract).

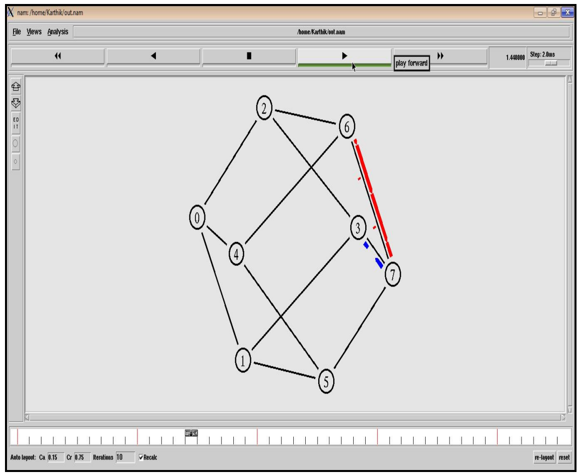

Fig. 1. Hypercube Interconnection Network

Folded Hypercube Network:

Folded Hypercube is an undirected graph formed from a hypercube graph by adding perfect matching edges that connects opposite pairs of hypercube vertices. The folded hypercube graph of order k (containing 2^{k-1} vertices) may be formed by adding edges between opposite pairs of vertices in a hypercube graph of order $k-1$. It can be formed from a hypercube graph (also) of order k, which has twice as many vertices, by identifying together (or contracting) every opposite pair of vertices. An order-k folded cube graph is k-regular with 2^{k-1} vertices and $2^{k-2}k$ edges. The chromatic number of the order-k folded cube graph is two when k is even (that is, in this case, the graph is bipartite) and four when k is odd. The odd girth of a folded cube of odd order is k, so for odd k greater than three the folded cube graphs provide a class of triangle-free graphs with chromatic number four and arbitrarily large odd girth. As a distance-regular graph with odd girth k and diameter $(k-1)/2$, the folded cubes of odd order are examples of generalized odd graphs. When k is odd, the bipartite double cover of the order-k folded cube is the order-k cube from which it was formed [9, 21].

The below figure shows the illustration of Folded Hypercube Interconnection Network in Network simulator:

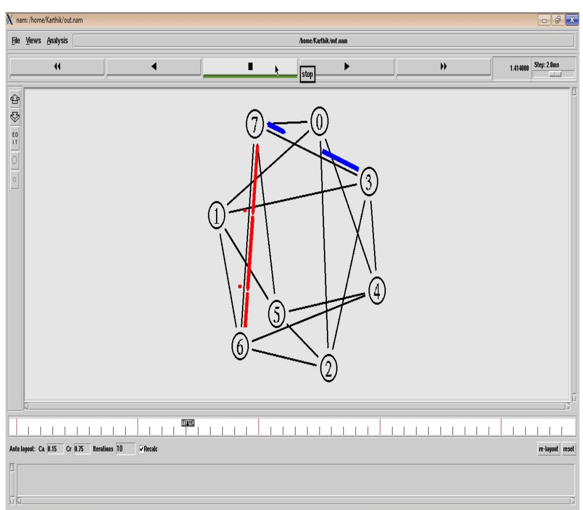

Fig. 2. Folded Hypercube Network

Multiply Twisted Cube Network:

An n -dimensional multiply-twisted hypercube Q_n has the same structural complexity as n-dimensional hypercube Q. That is, it has the same number of nodes and links, and each node has the same degree n, as Q_n. However, previous investigations indicate that due to some of its properties better than hypercube, the multiply-twisted hypercube is a good alternative for constructing multiprocessor systems. It is known that hypercube machines can simulate many multiprocessor systems based on other topologies such as trees, meshes, linear arrays and rings.

An ***n*** **-**dimensional multiply twisted cube has the same structural complexity as *n* **-**dimensional hypercube. That is they have the same number of nodes and links, and each node has the same degree n. However, previous investigations indicate that the multiply-twisted hypercube has some properties better than that of hypercube. The multiply-twisted hypercube is recursively defined, and it has a relative structure. It has observed that the diameter of *Q is [n+1]/2* which is about half of the diameter *n* of the n-dimensional hypercube Q. In addition, the average distance between nodes in *Q* is about 3/4 of the average distance between nodes in *Q*. In conjunction with the regularity, these properties can be used to design simple data communication algorithms for Q_n that are more efficient than those for conventional hypercube *Q*.

Also, many efficient hypercube algorithms can be directly modified to fit the twisted-hypercube without simulations by embedding so that undesirable overheads in such simulations can be avoided. It has been conjectured that the ***(2"*** **-**1)-node complete binary tree is a sub-graph of ***n*** **-**dimensional multiply-twisted hypercube (which has 2" nodes); but this is not true for the ***n*** dimensional hypercube. Since there is more and more evidence that multiply-twisted hypercube is a good alternative for the hypercube, it is important to further investigate the combinatorial structure and computational aspect of this architecture [1, 21].

The below figure shows the illustration of Multiply Twisted Cube Network in Network simulator:

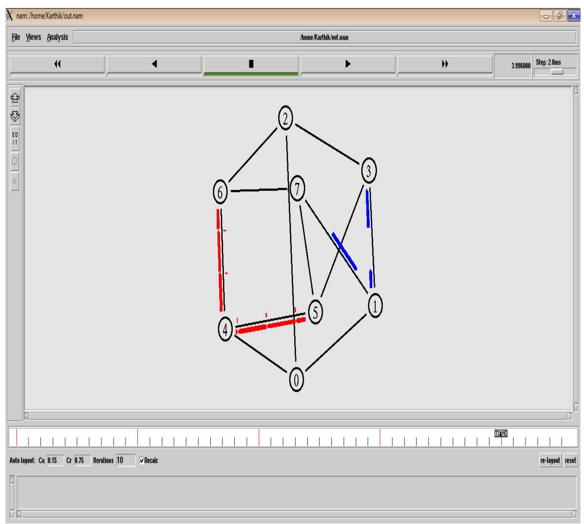

Fig. 3. Multiply Twisted Cube Network

Recursive Circulant Hypercube Network:
Recursive Circulant Hypercube Network considered in this paper is the recursive circulant graph G(N, d) proposed by Park and Chwa [19]

Recursive Circulant Hypercube Network similar to Recursive Circulant Graph G(N, d) is defined to be a Circulant graph with N nodes and jumps of powers of d, d□2. G(N, d) is node symmetric, has a Hamiltonian cycle unless N□2, and can be recursively constructed.

Definition: For two positive integers N and d, the recursive circulant graph G(N, d) has the vertex set V = {0, 1, . . .,N − 1}, and two vertices u and v are adjacent if and only if $u-v = \pm d^i$ (mod N) for some $0 \leq i \leq [\log_d N]-1$. The family of recursive circulant graphs is proposed as a network topology for multicomputer systems [17]. Recursive circulant graph G(N, d) is known to have a recursive structure for $N = d^m$, $m \geq 1$ and $N = cd^m$, $2 \leq c < d$, $m \geq 0$ induced by vertices {v | v ≡ j (mod d)} [18]. Furthermore, the edges not contained in any G_j form a Hamiltonian cycle. We call the Hamiltonian cycle with the edges of the form {i, i + 1}, i = 0, 1.

Cycles in networks are useful in applications such as embedding linear arrays and rings. We call a graph G with n vertices pancyclic if G contains cycles of every length k, $3 \leq k \leq n$. Since bipartite graphs have no odd cycles, a bipartite graph G is called bipancyclic if G has cycles of every even length. It is known that G(2m, 4), a special case of recursive circulant graphs, is pancyclic [18, 21].

The below figure shows the illustration of Recursive Circulant Hypercube Network in Network simulator:

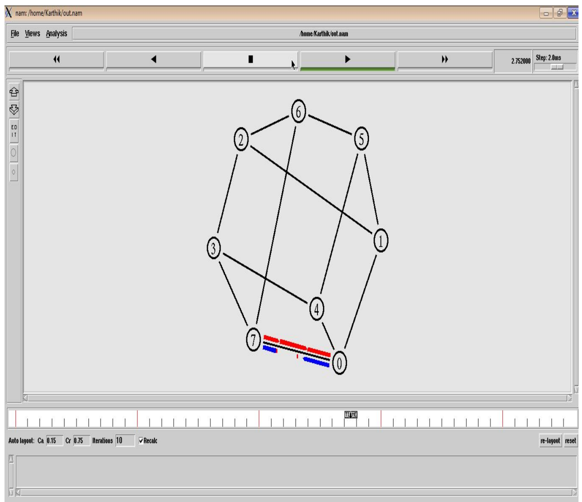

Fig. 4. Recursive Circulant Hypercube Network

Multiple Reduced Hypercube Network:

According to [1], the nodes of a Multiple Reduced Hypercube MRH(n) are expressed as n bit strings $s_n\, s_{n-1}...s_i...s_2\, s_1$ consisting of binary numbers {0,1} ($1 \leq i \leq n$). The edges of MRH(n) are expressed in three forms according to connection method, they are called hypercube edge, exchange edge, and complement edge, respectively, and are indicated as h-edge, x-edge, and c-edge, respectively. Each edge is defined into when n is an even number and n is an odd number.

Case 1: When n is an even number, it is assumed that for edge definition, $s_n\, s_{n-1}\, ...s_{i+1}$ is α and a bit string $s_i\, ...s_2\, s_1$ is β in the bit string of a node U(=$s_n\, s_{n-1}\, ..s_i\, ...s_2\, s_1$). Therefore the bit string of a node U(=$s_n\, s_{n-1}\, ..s_i\, ...s_2\, s_1$) can be simply expressed as αβ.
Assuming that the nodes U and V are adjacent with each other, adjacent edges are as follows. i) Hypercube edge : This edge indicates an edge linking two nodes U(=$s_n\, s_{n-1}\, ..s_i\, ...s_2\, s_1$) and V(=$s_n\, s_{n-1}\, ...s_{i+1}\, s_i\, ...s_2\, s_1$) of MRH(n) ($n/2 \leq j \leq n$).
ii) Exchange edge : This edge indicates an edge linking two nodes U(=αβ) and V(=βα) of MRH(n) if α≠β in the bit string of the nodes.
iii) Complement edge : This edge indicates an edge linking two nodes U(=s_n α'β') and V(=s_n α'β') of MRH(n) if α≠β in the bit string of the nodes.

Case 2: When n is an odd number, it is assumed that for edge definition, $s_{n-1}...s_i+1$ is α’ and a bit string $s_i...s_2\, s_1$ is β’ in the bit string of a node U(=$s_n\, s_{n-1}...s_i...s_2\, s_1$). Therefore a node U can be indicated as U (=s_n α'β')
i) Hypercube edge: This edge indicates an edge linking two nodes U (=snsn-1....sj...si+1si...s2s1) and V(=$s_n\, s_{n-1}...\, s_j\, ...s_{i+1}\, s_i...s_2\, s_1$) of MRH(n)
ii) Exchange edge: This edge indicates an edge linking two nodes U(=s_n α'β') and V(=s_n β’α’) of MRH(n) in the bit string of a node.
iii) Complement edge: This edge indicates an edge linking two nodes U(=s_n α'β') and V(=s_n α'β') of MRH(n) if α'= β' in the bit string of a node.

Node (edge) connectivity is the least number of nodes (edges) that are required to be eliminated to divide an interconnection network into two or more parts without common nodes [2, 21].

The below figure shows the illustration of Multiple Reduced Hypercube Network in Network simulator:

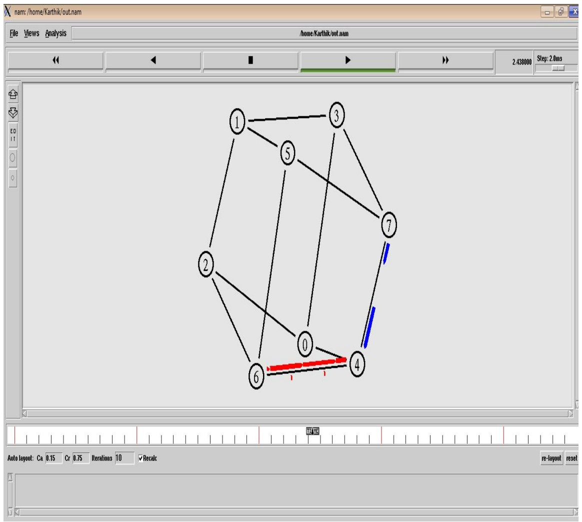

Fig. 5. Multiple Reduced Hypercube Network

III. DISCUSSION AND CONCLUSION

In this paper, different variants of interconnection networks such as Hypercube H(n), Folded Hypercube FH(n), Multiply twisted Cube MTC(n), Recursive Circulant RC(n) and Multiple Reduced Hypercube MRH(n) are illustrated in Network Simulator 2 and various physical and logical properties are analyzed to summarize the differences in their applications.

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